PS8.A: Preindustrial CH_4 Emission Rate

Preindustrial concentration of was 700 ppbv; residence time is 12 years. What was the preindustrial emission rate of methane, in Tg/y?

$$F = \frac{S}{\tau} = \frac{cV}{\tau}$$

$$= \frac{\left(700 \cdot 10^{-9} \frac{\text{mole}_{CH_4}}{\text{mole}_{air}}\right)\left(1.8 \cdot 10^{20}\ \text{mole}_{air}\right)\left(\frac{16\ g_{CH_4}}{\text{mole}_{CH_4}}\right)\left(\frac{Tg}{10^{12}\ g}\right)}{12\ y}$$

$$= 168 \cong 170\ \frac{Tg_{CH_4}}{y}$$

PS8.B/C: Equilibrium CH_4 Concentration

Anthropogenic emissions are estimated at 350 Tg/y. If anthropogenic and natural emissions remain constant, what will be the equilibrium concentration of methane?

$$c = \frac{\tau F}{V} \qquad \frac{c_1}{c_0} = \frac{\tau F_1}{\tau F_0}$$

$$c_1 = c_0 \frac{F_1}{F_0} = (700\ \text{ppb}) \frac{168\ Tg + 350\ Tg}{168\ Tg} = 2158 \approx 2200\ \text{ppb}$$

Compare to 1998 concentration of 1745 ppb. It would take about three residence times--about 40 years--to reach equilibrium.

PS8.D: CH_4 Radiative Forcing

Radiative forcing due to the increase in methane from the preindustrial to the 1998 concentration is 0.48 W/m^2. Additional increases add 3.7×10^{-4} W/m^2 per ppbv.

$$\Delta F = \left(0.48 \frac{W}{m^2}\right) + (2158\ \text{ppb} - 1745\ \text{ppb})\left(3.7 \cdot 10^{-4}\ \frac{W}{m^2 \text{ppb}}\right)$$

$$= 0.48 \frac{W}{m^2} + (413\ \text{ppb})\left(3.7 \cdot 10^{-4}\ \frac{W}{m^2 \text{ppb}}\right)$$

$$= 0.48 \frac{W}{m^2} + 0.15 \frac{W}{m^2} = 0.63 \frac{W}{m^2}$$

PS8.E: Preindustrial N_2O Emission Rate

Preindustrial concentration of N_2O was 270 ppbv; residence time is 114 years. What was the preindustrial emission rate, in Tg/y?

$$F = \frac{S}{\tau} = \frac{cV}{\tau}$$

$$= \frac{\left(270 \cdot 10^{-9} \frac{\text{mole}_{N_2O}}{\text{mole}_{air}}\right)\left(1.8 \cdot 10^{20}\ \text{mole}_{air}\right)\left(\frac{44\ g_{N_2O}}{\text{mole}_{N_2O}}\right)\left(\frac{Tg}{10^{12}\ g}\right)}{114\ y}$$

$$= 18.8 \cong 19\ \frac{Tg_{N_2O}}{y}$$

PS8.E: Equilibrium N_2O Concentration

Anthropogenic emissions are estimated at 11 Tg/y. If anthropogenic and natural emissions remain constant, what will be the equilibrium concentration?

$$c = \frac{\tau F}{V} \qquad \frac{c_1}{c_0} = \frac{\tau F_1}{\tau F_0}$$

$$c_1 = c_0 \frac{F_1}{F_0} = (270\text{ ppb}) \frac{18.8\text{ Tg} + 11\text{ Tg}}{18.8\text{ Tg}} = 428 \approx 430\text{ ppb}$$

Compare to 1998 concentration of 214 ppb. It would take roughly three residence times--about 400 years--to reach equilibrium.

PS8.E: N_2O Radiative Forcing

Radiative forcing due to the increase in N_2O from the preindustrial to the 1998 concentration is 0.15 W/m^2. Additional increases add 3.1 x 10^{-3} W/m^2 per ppbv.

$$\Delta F = \left(0.15 \frac{W}{m^2}\right) + (428\text{ ppb} - 314\text{ ppb})\left(3.1 \cdot 10^{-3} \frac{W}{m^2 ppb}\right)$$

$$= 0.15 \frac{W}{m^2} + (114\text{ ppb})\left(3.1 \cdot 10^{-3} \frac{W}{m^2 ppb}\right)$$

$$= 0.15 \frac{W}{m^2} + 0.35 \frac{W}{m^2} = 0.50 \frac{W}{m^2}$$

PS8.F: Stablizing CO_2 Concentrations

10% of CO_2 is retained with τ = 2 y, 25% with τ = 17 y, 30% with τ = 75 y, remainder with τ = 400 y. If emissions are 7 Pg_C/y, what would be the equilibrium concentration?

$$S = \text{natural stock} + \text{added stock} = S_0 + \Delta S$$

$$\Delta S = \tau F = \tau_1 F_1 + \tau_2 F_2 + \tau_3 F_3 + \tau_4 F_4 = (f_1\tau_1 + f_2\tau_2 + f_3\tau_3 + f_4\tau_4)F$$

$$= [0.1(2\text{ y}) + 0.25(17\text{ y}) + 0.3(75\text{ y}) + 0.35(400\text{ y})] \frac{7\text{ Pg}_C}{y}$$

$$= (167\text{ y})\left(\frac{7\text{ Pg}_C}{y}\right)\left(\frac{10^{15}\text{ g}}{Pg}\right) = 1169 \cdot 10^{15}\text{ g}_C$$

PS8.F: Stablizing CO_2 Concentrations

$$c = \frac{S}{V} = \frac{\Delta S + S_0}{V} = \frac{\Delta S}{V} + c_0 = \frac{(1169 \cdot 10^{15}\text{ g}_C)\left(\frac{\text{mole}_{CO_2}}{12\text{ g}_C}\right)}{1.8 \cdot 10^{20}\text{ mole}_{air}} + c_0$$

$$= 5.41 \cdot 10^{-4} + c_0 = 541\text{ ppm} + 278\text{ ppm} = 819 \approx 820\text{ ppm}$$

$$\Delta F = 5.3 \log_e\left(\frac{c}{c_0}\right) = 5.3 \log_e\left(\frac{819}{278}\right) = 5.77 \frac{W}{m^2}$$

PS8.G/H: Total ΔF, ΔT

$$\Delta F = \Delta F_{CO_2} + \Delta F_{CH_4} + \Delta F_{N_2O}$$

$$= 5.77 + 0.63 + 0.50 = 6.90\,\frac{W}{m^2}$$

$$c_{CO_2} = c_0 e^{\frac{\Delta F}{5.3}} = (278\text{ ppm})\, e^{\frac{6.9}{5.3}} = 1013 \approx 1000\text{ ppm}$$

$$\Delta T = \Delta F\left(\frac{\Delta T_{2X}}{\Delta F_{2X}}\right) = \left(6.9\,\frac{W}{m^2}\right)\left(\frac{2.5\ ^\circ C}{3.7\,\frac{W}{m^2}}\right) = 4.7\ ^\circ C$$

With a range of 2.8 to 8.4 ºC, for T_{2X} = 1.5 to 4.5 ºC.