PS4.1: Exhausting Fossil Fuels

$$T_{oil} = \frac{\ln\left[1+\frac{rM(0)}{F(0)}\right]}{r} = \frac{\ln\left[1+\frac{0.0162(10\cdot 10^{21})}{158\cdot 10^{18}}\right]}{0.0162} = 43.6 \approx 40\ y$$

$$T_{gas} = \frac{\ln\left[1+\frac{0.0254(10\cdot 10^{21})}{101\cdot 10^{18}}\right]}{0.0254} = 49.5 \approx 50\ y$$

$$T_{coal} = \frac{\ln\left[1+\frac{0.0240(250\cdot 10^{21})}{116\cdot 10^{18}}\right]}{0.0240} = 165 \approx 200\ y$$

PS4.2A: Mirage of Growing Supply

$$T = \frac{\ln\left[1+\frac{rM(0)}{F(0)}\right]}{r}$$

$$r = \ln(1+i) = \ln(1+0.05) = 0.0488$$

100-y supply lasts 36 y
1,000-y supply lasts 80 y
10,000-y supply lasts 127 y

$$T_{100} = \frac{\ln[1+0.0488(100)]}{0.0488} \cong 36y$$

$$T_{1000} = \frac{\ln[1+0.0488(1000)]}{0.0488} \cong 80\ y$$

$$T_{10000} = \frac{\ln[1+0.0488(10000)]}{0.0488} \cong 127\ y$$

PS4.2B: Mirage of Growing Supply

"Doubling the size of the oil reserve will add at most 14 years to the life expectancy of the resource if we continue to use it at the currently increasing rate, no matter how large it is currently."

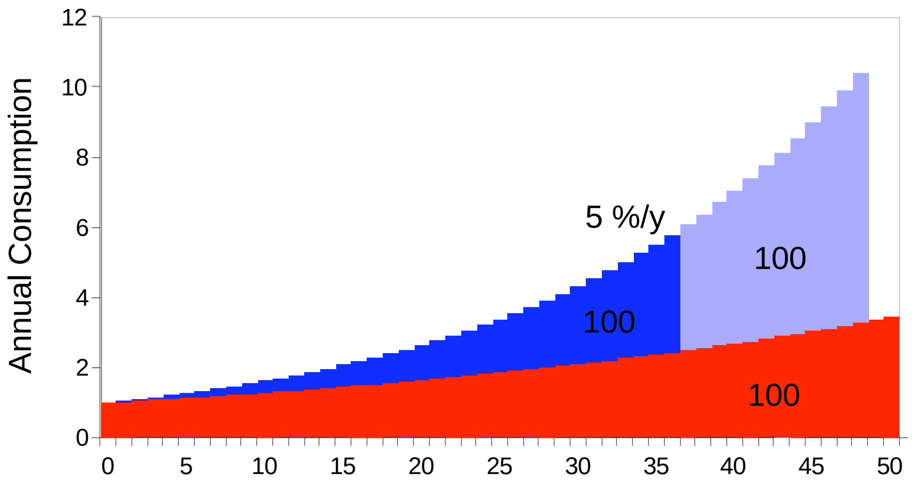
$\tau = M_0/F_0$	T_τ	$T_{2\tau}$	$T_{2\tau} - T_\tau$
100	36	49	12.4
1,000	80	94	14.0
10,000	127	141	14.2

PS4.2B: Mirage of Growing Supply

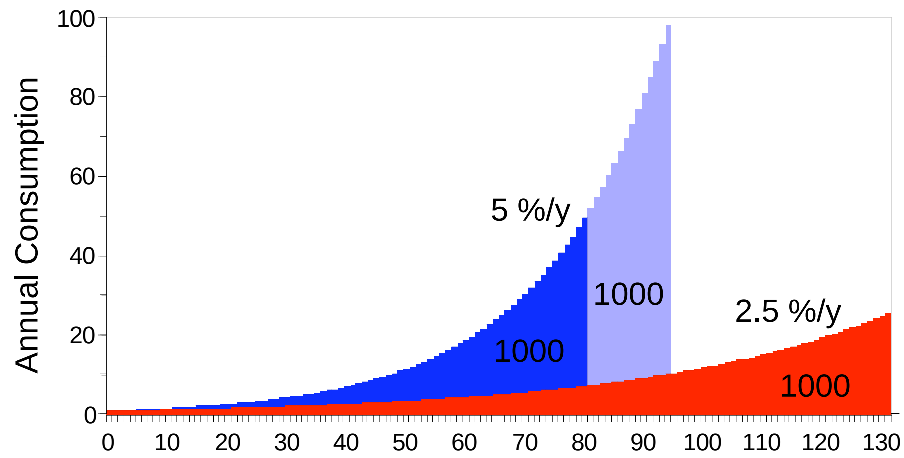
"On the other hand, halving the growth of consumption will almost double the life expectancy of the supply, no matter what it is."

$\tau = M_0/F_0$	T_r	$T_{r/2}$	$T_{r/2} / T_r$
100	36	51	1.39
1,000	80	133	1.66
10,000	127	225	1.78

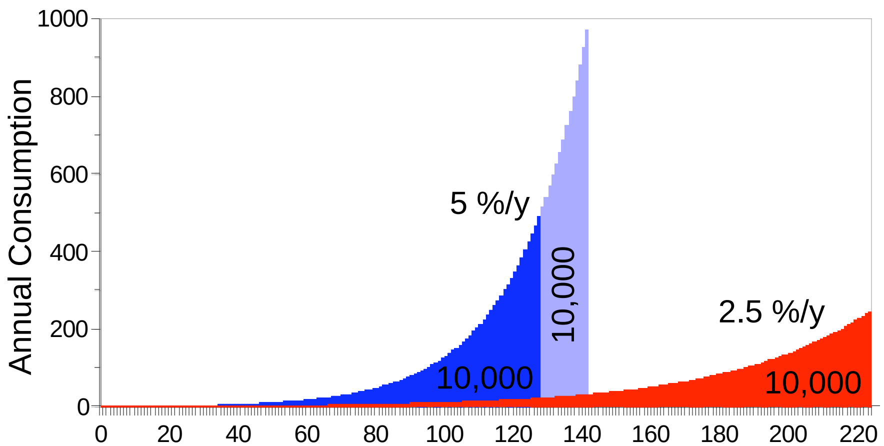
Doubling Supply v. Halving Growth Rate
M/F = 100



Doubling Supply v. Halving Growth Rate
M/F = 1,000



Doubling Supply v. Halving Growth Rate
M/F = 10,000



PS4.2B: Mirage of Growing Supply

In general, if the stock increases by a factor of n:

$$T_\tau = \frac{\ln[1+r\tau]}{r} \qquad T_{n\tau} = \frac{\ln[1+r(n\tau)]}{r}$$

$$T_{n\tau} - T_\tau = \frac{1}{r}[\ln(1+nr\tau) - \ln(1+r\tau)] = \frac{\ln\left[\frac{1+nr\tau}{1+r\tau}\right]}{r}$$

$$\text{if } \; r\tau \gg 1 \qquad \frac{\ln\left[\frac{1+nr\tau}{1+r\tau}\right]}{r} \cong \frac{\ln\left[\frac{nr\tau}{r\tau}\right]}{r} = \frac{\ln(n)}{r}$$

$$T_{2\tau} - T_\tau \cong \frac{\ln(2)}{r} = \frac{0.693}{0.0488} = 14.2\text{ y}$$

PS4.2B: Mirage of Growing Supply

If the stock is 10 times greater (n = 10), then:

$$\text{if} \quad n = 10 \qquad T_{10\tau} - T_{\tau} \cong \frac{\ln(10)}{r} = \frac{2.30}{0.0488} = 47 \text{ y}$$

100-y supply lasts	36 y
1,000-y supply lasts	80 y
10,000-y supply lasts	127 y

44 y

47 y

PS4.2B: Mirage of Growing Supply

In general, if growth rate decreases by a factor of n:

$$T_r = \frac{\ln[1+r\tau]}{r} \qquad T_{r/n} = \frac{\ln[1+r\tau/n]}{r/n} \qquad \frac{T_{r/n}}{T_r} = n\frac{\ln[1+r\tau/n]}{\ln[1+r\tau]}$$

$$\text{if} \quad r\tau \gg 1 \qquad \frac{T_{r/n}}{T_r} \cong n\frac{\ln\left[\frac{r\tau}{n}\right]}{\ln[r\tau]} = n\frac{\ln(r\tau)-\ln(n)}{\ln(r\tau)} = n\left[1-\frac{\ln(n)}{\ln(r\tau)}\right]$$

$$\text{if} \quad n = 2 \qquad \frac{T_{r/2}}{T_r} = 2\left[1-\frac{\ln(2)}{\ln(r\tau)}\right]$$

compare to 1.66 above

$$\text{if} \quad r\tau = (0.0488)(1000) = 48.8 \qquad \frac{T_{r/2}}{T_r} = 2\left[1-\frac{\ln(2)}{\ln(48.8)}\right] = 1.64$$

PS4.3A/B: Fuel Efficiency

	VMT (millions)			Fuel (million gallons)			Average miles per gallon		
	Cars	Trucks	Total	Cars	Trucks	Total	Cars	Trucks	Total
1994	1,406,000	765,000	2,171,000	67,874	44,112	111,986	20.7	17.3	19.4
2004	1,704,982	1,014,342	2,719,324	76,007	62,626	138,632	22.4	16.2	19.6

$$\left[\frac{\text{gal}}{19.4 \text{ mi}}\right]\left[\frac{\text{mi}}{1.609 \text{ km}}\right]\left[\frac{3.785 \text{ L}}{\text{gal}}\right] = 0.121\frac{\text{L}}{\text{km}} = 12.1\frac{\text{L}}{100 \text{ km}}$$

$$\left[\frac{\text{gal}}{19.6 \text{ mi}}\right]\left[\frac{\text{mi}}{1.609 \text{ km}}\right]\left[\frac{3.785 \text{ L}}{\text{gal}}\right] = 0.120\frac{\text{L}}{\text{km}} = 12.0\frac{\text{L}}{100 \text{ km}}$$

PS4.3C/D: Gasoline Savings

$$\left[\frac{2719\cdot 10^9 \text{ mi}}{\text{y}}\right][20 \text{ y}]\left[\frac{\text{gal}}{19.6 \text{ mi}}\right] = 2773\cdot 10^9 \text{ gal}$$

$$-\left[\frac{2719\cdot 10^9 \text{ mi}}{\text{y}}\right][20 \text{ y}]\left[\frac{\text{gal}}{24.6 \text{ mi}}\right] = \underline{2209\cdot 10^9 \text{ gal}}$$

$$563\cdot 10^9 \text{ gal}$$

ANWR would yield ~200 Ggal over 20 y; to save this much gas, 2773 – 200 = 2573 Ggal would be consumed over 20 y:

$$\frac{\left[\frac{2719\cdot 10^9 \text{ mi}}{\text{y}}\right][20 \text{ y}]}{2573\cdot 10^9 \text{ gal}} = 21.1\frac{\text{mi}}{\text{gal}} - 19.6\frac{\text{mi}}{\text{gal}} = 1.5\frac{\text{mi}}{\text{gal}}$$

PS4.3E/F: Growth Rate, VMT

$$r = \frac{\ln\left(\frac{2719 \cdot 10^9\ \text{mi}}{2171 \cdot 10^9\ \text{mi}}\right)}{10\ \text{y}} = \frac{0.0225}{\text{y}} = 2.25\,\frac{\%}{\text{y}}$$

$$i = \left(\frac{2719 \cdot 10^9\ \text{mi}}{2171 \cdot 10^9\ \text{mi}}\right)^{\frac{1}{10}} - 1 = 0.0228 = 2.28\,\frac{\%}{\text{y}}$$

$$\text{VMT}_{2015-2035} = \frac{\text{VMT}_{2004}}{r}\left[e^{r(2035-2004)} - e^{r(2015-2004)}\right]$$

$$= \frac{2719 \cdot 10^9\ \text{mi}}{0.0225}\left[e^{0.0225(31)} - e^{0.0225(11)}\right]$$

$$= 88{,}011 \cdot 10^9\ \text{mi}$$

PS4.3G/H: Revised Savings

$$\left[88011 \cdot 10^9\ \text{mi}\right]\left[\frac{\text{gal}}{19.6\ \text{mi}}\right] = 4487 \cdot 10^9\ \text{gal}$$

$$-\left[88011 \cdot 10^9\ \text{mi}\right]\left[\frac{\text{gal}}{24.6\ \text{mi}}\right] = \underline{3575 \cdot 10^9\ \text{gal}}$$

$$911 \cdot 10^9\ \text{gal}$$

$$\frac{88011 \cdot 10^9\ \text{mi}}{4287 \cdot 10^9\ \text{gal}} = 20.5\,\frac{\text{mi}}{\text{gal}} - 19.6\,\frac{\text{mi}}{\text{gal}} = 0.9\,\frac{\text{mi}}{\text{gal}}$$