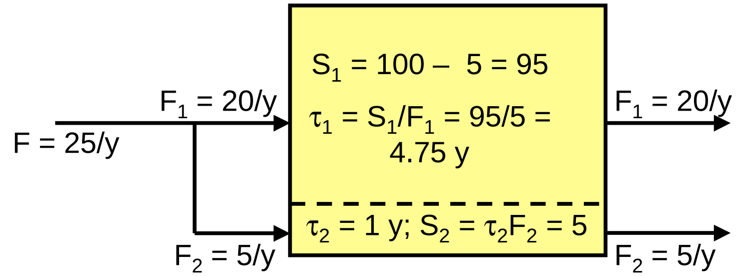
PS3.1: School as Steady-State System

1. $$S = F\tau = \left[\frac{7\text{ calves}}{\text{y}}\right]\left[6\text{ y}\right] = 42\text{ cows}$$

2a. $$\tau = \frac{S}{F} = \frac{100\text{ enrolled students}}{25\,\frac{\text{entering students}}{\text{y}}} = 4\text{ y}$$

2b. $$\tau = \frac{S}{F} = \frac{95\text{ students who will graduate}}{20\,\frac{\text{graduating students}}{\text{y}}} = 4.75\text{ y}$$

PS3.1.2b: A Visual Presentation



PS3.2: The Water Above

We will assume the ocean to be in steady state. It is then easier to work with outflow because we are then dealing with only evaporation.

$$\tau = \frac{S}{F} = \frac{1350\cdot 10^{15}\text{ m}^3}{456\cdot 10^{12}\,\frac{\text{m}^3}{\text{y}}} = 2960\text{ y} \approx 3000\text{ y}$$

PS3.3: A Polluted Lake

Lake volume = 10,000 ac-ft, outflow = 1,000 ac-ft/y. Hg concentration = 1 ppm. Mercury flow into lake?

$$\tau = \frac{10{,}000\text{ ac-ft}}{1{,}000\,\frac{\text{ac-ft}}{\text{y}}} = 10\text{ y}$$

$$S = cV = \left[\frac{1\text{ t}_{Hg}}{10^6\text{ t}_{H_2O}}\right]\left[10^4\text{ ac-ft}\right]\left[\frac{1234\text{ m}^3}{\text{ac-ft}}\right]\left[\frac{1\text{ t}_{H_2O}}{\text{m}^3}\right] = 12\text{ t}_{Hg}$$

$$F = \frac{S}{\tau} = \frac{12\text{ t}_{Hg}}{10\text{ y}} = 1.2\,\frac{\text{t}_{Hg}}{\text{y}} \approx 3\,\frac{\text{kg}_{Hg}}{\text{d}}$$

PS3.4: Paper in Landfill

Of the 5 lb/d waste that is generated, 2 lb is paper. Of this only 0.8 lb/d ends up in landfill.

$$\text{Flow of paper waste into landfill} = F = 0.8\ \frac{\text{lb}}{\text{d}}$$

$$\text{Residence time} = \tau = 10\ \text{y} = 3650\ \text{d}$$

$$S = F\tau = \left[\frac{0.8\ \text{lb}}{\text{person}\cdot\text{d}}\right]\left[3650\ \text{d}\right] \approx 3{,}000\frac{\text{lb}}{\text{person}}$$

PS3.5: Motor Oil in Chesapeake

a. From PS1, 20 Mgal/y is released; if half flows into the Bay, $F_{IN} \approx 10$ Mgal/y.

b. Let's assume oil is mixed enough in the bay with a residence time of 1 y.

$$S = F\tau = \left[\frac{10^7\ \text{gal}}{\text{y}}\right]\left[\frac{\text{m}^3}{264\ \text{gal}}\right]\left[\frac{0.8\ \text{t}}{\text{m}^3}\right]\left[1\ \text{y}\right] = 30{,}000\ \text{t}$$

c. Volume of the bay is 68 km^3

$$c = \frac{S}{V} = \frac{3\cdot 10^4\ t_{oil}}{\left[68\ \text{km}^3\right]\left[\frac{10^9\ \text{m}^3}{\text{km}^3}\right]\left[\frac{t_{H_2O}}{\text{m}^3}\right]} = 4.5\cdot 10^{-7}\frac{t_{oil}}{t_{H_2O}}$$

$$\approx 0.5\ \text{ppm}$$

PS3.6: Average Growth Rates

BP gives oil consumption of 3204 Mtoe in 1994 and 3767 in 2004; the average growth rate of consumption:

$$r = \frac{\ln\left(\frac{S_{2004}}{S_{1994}}\right)}{2004-1994} = \frac{\ln\left(\frac{3767\ \text{Mtoe / y}}{3204\ \text{Mtoe / y}}\right)}{10\ \text{y}} = 0.0162 \cong 1.6\frac{\%}{\text{y}}$$

$$i = \left(\frac{S_{2004}}{S_{1994}}\right)^{\frac{1}{10}} - 1 = \left(\frac{3767}{3204}\right)^{\frac{1}{10}} - 1 = 0.0163 \approx 1.6\frac{\%}{\text{y}}$$

BP: World Consumption, Mtoe

	1994	**2004**	**r**	**i**
Oil	3204	3767	1.62%	1.63%
Coal	2186	2778	2.40%	2.43%
Gas	1877	2420	2.54%	2.58%

PS3.7: U.S. GDP Growth

Per-capita GDP in 2000 = $34,000/y, r = 2 %/y

A. How many years does it take for income to double?

B. Estimate income in 2100 at this growth rate

$$T_{2X} = \frac{\ln 2}{r} = \frac{0.69}{0.02} = 34.5 \approx 35\ y$$

$$100\ y \approx 3\ \text{doublings} \Rightarrow 2 \times 2 \times 2 \times \frac{\$34,000}{y} = \frac{\$270,000}{y}$$

$$F_{2100} = F_{2000} e^{r(2100-2000)} = (34000)e^{2} = 251{,}228 \approx \frac{\$250,000}{y}$$

PS3.8: China GDP Growth

$$i = e^{0.08} - 1 = 1.0833 - 1 = 0.0833 = 8.3\ \%/y$$

In 2000, per-capita GDP in China was $4,000/y, increasing at an average rate of 8 %/y.

A. If r = 0.08/y, what is i?

B. If growth continues at this rate, when will per-capita GDP in China equal that in the US?

$$F_{US}(0) = F_{CH}(0)\, e^{r_{CH}T}$$

$$\frac{F_{US}(0)}{F_{CH}(0)} = e^{r_{CH}T}$$

$$T = \frac{\ln\left(\frac{F_{US}(0)}{F_{CH}(0)}\right)}{r_{CH}} = \frac{\ln\left(\frac{34000}{4000}\right)}{0.08} = 26.8 \approx 27\ y$$

PS3.8: China GDP Growth

If per-capita GDP continues to grow at 8 percent per year in China and 2 percent per year in the United States, in what year would China catch up to the United States? What would the per-capita income be in both countries at that time?

$$F_{US}(0)\, e^{r_{US}T} = F_{CH}(0)\, e^{r_{CH}T}$$

$$\frac{F_{US}(0)}{F_{CH}(0)} = \frac{e^{r_{CH}T}}{e^{r_{US}T}} = e^{(r_{CH}-r_{US})T}$$

$$T = \frac{\ln\left(\frac{F_{US}(0)}{F_{CH}(0)}\right)}{r_{CH} - r_{US}} = \frac{\ln\left(\frac{34000}{4000}\right)}{0.08 - 0.02} = \frac{\ln(8.5)}{0.06} = 35.7 \approx 36\ y = 2036$$

$$F_{CH}(2036) = (4000)e^{0.08(35.7)} = 69388 \approx \frac{\$70,000}{y}$$