MID-TERM EXAM SOLUTION

1. A. The human population is projected to increase by 50 percent over the next 50 years, from 6 to 9 billion people. What average annual growth rate does this represent?

$$\mathbf{i = \left(\frac{9}{6}\right)^{\frac{1}{50}} - 1 = 0.00814 = 0.8\%/y} \qquad \mathbf{r = \frac{\ln\left(\frac{9}{6}\right)}{50} = 0.00811 \cong 0.8\%/y}$$

B. Today, average income is about $5000 per person. If per-capita income increases at an average rate of 1 percent per year, what will average income be in 2050?

$$\mathbf{5000(1.01)^{50} = 8200} \qquad \mathbf{5000\,e^{0.01(50)} = 8240}$$

C. If global annual emissions of a particular pollutant (e.g., carbon dioxide) are to remain constant over this period, at what rate must technological efficiency (emissions per dollar of income) improve?

emissions = (population)·($/person)·(emissions/$) = (pop)·(inc)·(tech):

$r_{em} = r_{pop} + r_{inc} + r_{tech}$; if $r_{em} = 0$, then $r_{tech} = -(r_{pop} + r_{inc}) = -1.8\%/y$

2. A. A major source of indoor air pollution is the combustion product benzo[a]pyrene (BAP), which is classified by EPA as a probable human carcinogen. In many rural Chinese and Indian villages, coal is burned more or less continuously to boil water. Estimate the concentration of BAP (ng/m^3) in the indoor air, assuming 1 kg of coal per day is burned, that the stove releases 50 mg BAP per kg of coal burned, and that the average residence time of air in a small, two-room hut is about 10 minutes.

You first have to estimate the volume of the hut. I think 50 m^3 is a good estimate, but anything from 35 to 100 m^3 is acceptable.

$$\mathbf{c = \frac{S}{V} = \frac{F\tau}{V} = \frac{\left(\frac{1\,kg_{coal}}{d}\right)\left(\frac{50\,mg}{kg_{coal}}\right)\left(\frac{10^6\,ng}{mg}\right)(10\,min)\left(\frac{h}{60\,min}\right)\left(\frac{d}{24\,h}\right)}{(50\,m^3)} = 7000\,\frac{ng}{m^3}}$$

B. The EPA has estimated that the risk of lung cancer death from continuous exposure to BAP is $2 \cdot 10^{-6}$ per μg inhaled. If a person is exposed to the concentration of BAP calculated in part A continuously for 20 years, what would be the resulting risk of lung cancer death? Based on this calculation, do you judge that indoor air pollution is a major health risk in such situations?

Spherical Cow gives breathing rates of 19 and 20 L/min for women and men for light activity, and 6.0 and 7.5 L/min for resting. A very rough average for men and women and for time spent indoors might be 10 L/min. The total amount of BAP inhaled during 20 years is therefore

$$\left(\frac{10\text{ L}}{\text{min}}\right)\left(\frac{\text{m}^3}{1000\text{ L}}\right)\left(\frac{60\text{ min}}{\text{h}}\right)\left(\frac{24\text{ h}}{\text{d}}\right)\left(\frac{365\text{ d}}{\text{y}}\right)(20\text{ y})\left(\frac{7\ \mu\text{g}}{\text{m}^3}\right) = 740{,}000\ \mu\text{g}_{\text{BAP}}$$

$$\left(\frac{2 \cdot 10^{-6}}{\mu\text{g}_{\text{BAP}}}\right)(740{,}000\ \mu\text{g}_{\text{BAP}}) = 1.5$$

So the risk of cancer is greater than one! Sorry about that!

3. A economic model predicts that a tax equal to \$100 per metric ton of carbon is needed to meet the emission reductions required by the Kyoto Protocol.

A. Express this tax in dollars per barrel of oil, and as a percent increase over the current price of oil.

According to *Spherical Cow*, oil is 98% $CH_{1.5}$ by weight; $CH_{1.5}$ is (12)/(13.5) = 89% carbon; thus, oil is (0.98)(0.89) = 87% carbon. The density of oil is not given directly, but you should be able to guess that it is somewhat less than 1 t/m^3, or you could use the information on p. 242 of *Spherical Cow* for the energy content of crude oil:

$$\left(\frac{6.1 \cdot 10^9\text{ J}}{\text{bbl}}\right)\left(\frac{\text{kg}}{43 \cdot 10^6\text{ J}}\right) = 142\frac{\text{kg}}{\text{bbl}}$$

Which, by the way, is 890 kg/bbl.

$$\left(\frac{\$100}{\text{tC}}\right)\left(\frac{0.87\text{ tC}}{\text{t}_{\text{oil}}}\right)\left(\frac{0.142\text{ t}_{\text{oil}}}{\text{bbl}}\right) = \frac{\$12}{\text{bbl}}$$

Which you can compare to the current price of about \$35/bbl; thus, the carbon tax presents an increase of about 33% in the price of oil.

B. Express this tax in cents per kilowatt-hour of coal-fired electricity, and as a percent increase over the current retail price of electricity. Coal-fired power plants are about 35 percent efficient in converting heat to electricity.

According to *Spherical Cow*, coal is 75% $CH_{0.8}$ by weight; $CH_{0.8}$ is (12)/(12.8) = 94% carbon; thus, coal is (0.75)(0.94) = 70% carbon.

$$\left(\frac{\$100}{\text{tC}}\right)\left(\frac{0.7\ \text{tC}}{\text{t}_{\text{coal}}}\right)\left(\frac{\text{t}_{\text{coal}}}{29.3\cdot10^{9}\ \text{J}_{\text{th}}}\right)\left(\frac{\text{J}_{\text{th}}}{0.35\ \text{J}_{\text{e}}}\right)\left(\frac{\text{J}}{\text{W}\cdot\text{s}}\right)\left(\frac{10^{3}\ \text{W}}{\text{kW}}\right)\left(\frac{3600\ \text{s}}{\text{h}}\right)=\frac{\$0.025}{\text{kWh}}$$

I paid a total of \$0.097/kWh on my last Pepco bill, so the carbon tax would represent an increase of about 25% on the retail price, assuming all other costs remained the same. On the other hand, Pepco charges about \$0.05/kWh for the electricity (the rest is transmission, distribution, and various taxes), so the carbon tax would increase the energy charge by 50%.

4. Mean sea level rose by about 15 centimeters from 1890 to 1990. Possible sources include melting of glaciers and ice sheets, thermal expansion of seawater, and excess withdrawals of groundwater. Estimate the increase due to the latter source. In 1990, global water withdrawals were 3500 cubic kilometers, of which roughly half was from groundwater. From 1940 to 1990, groundwater withdrawals increased at a rate of about 3 percent per year. About 70 percent of withdrawn groundwater is used for agriculture; the remainder is used for domestic and industrial supply.

In 1990, groundwater withdrawals were approximately (3500/2) = 1750 km^3/y. If withdrawal increased at a constant rate r, the amount withdrawn in year t is

$$F(t)=1750\,e^{r(t-1990)} \qquad F(1940)=1750\,e^{\left(\frac{0.03}{y}\right)(-50\,y)}=390\ \text{km}^3/\text{y}$$

We need the total amount of groundwater withdrawn. One approach would simply be to take the average of these two: [(1750 + 390)/2][50 y] = 54,000 km^3/y; this overestimates the amount withdrawn from 1940–90, but ignores the amount withdrawn before 1940. Another would be to calculate each year and add them all up (e.g., using Excel). A better way would be to use calculus. In problem II.22, Harte finds the total amount withdrawn, M, over T years as:

$$e^{rT}=1+\frac{rM}{F} \qquad M=\frac{F}{r}\left(e^{rT}-1\right)$$

where r is the rate of increase and F is the rate of withdrawal at the beginning of the period. In our case, r = 0.03, T = 100 y, and F = $1750e^{-3}$ = 87 km^3/y.

$$M = \frac{\left(\frac{87\ \text{km}^3}{y}\right)}{\left(\frac{0.03}{y}\right)}\left(e^{\left(\frac{0.03}{y}\right)(100\ y)} - 1\right) = 55{,}000\,\text{km}^3$$

If all this water was added to the ocean, sea level would rise by

$$\left(\frac{55{,}000\ \text{km}^3}{360 \cdot 10^6\ \text{km}^2}\right)\left(\frac{10^5\ \text{cm}}{\text{km}}\right) = 15\,\text{cm}$$

which is equal to the total increase in sea level. Of course, not all of the withdrawn groundwater flowed into the ocean. If we assume that only the 30% withdrawn for domestic and industrial supply was discharged into the oceans, then the rise due to groundwater discharge would be (0.3)(15 cm) = 5 cm, which is the "missing" sea level rise.