

FINAL EXAM SOLUTIONS

1. Coal consumption in India was 3309 and 6140 PJ/yr in 1983 and 1993, respectively.

A. Estimate the average annual growth rate of coal consumption. (5 pts)

$$(1 + i)^{10} = (6140/3309); \quad i = (6140/3309)^{1/10} - 1 = 0.064 = 6.4\%/yr \text{ annual}$$

$$e^{10r} = (6140/3309); \quad r = \ln(6140/3309)/10 = 0.062 = 6.2\%/yr \text{ continuous}$$

B. Indian coal has an average energy content of 26 MJ/kg and an average sulfur content of 2 percent by weight. How much sulfur was emitted by Indian coal burning in 1993, in Mte? Compare this to the 9 Mte(S) emitted by the United States in the same year. (5 points)

$$\left[\frac{6140 \cdot 10^{15} \text{ J}}{\text{yr}} \right] \left[\frac{\text{kg}(\text{coal})}{26 \cdot 10^6 \text{ J}} \right] \left[\frac{0.02 \text{ kg}(\text{S})}{\text{kg}(\text{coal})} \right] \left[\frac{\text{te}}{10^3 \text{ kg}} \right] \left[\frac{\text{Mte}}{10^6 \text{ te}} \right] = 4.7 \text{ Mte}(\text{S})$$

The U.S. (with 3.5 times fewer people) emitted about twice as much sulfur.

C. How much sulfur would India emit in 2013 if the consumption of coal continues to grow at the 1983-93 rate? (5 points)

$$(4.72 \text{ Mte}(\text{S})) \cdot (1.064)^{20} = 16.3 \text{ Mte}(\text{S}) \quad (4.72 \text{ Mte}(\text{S})) \cdot e^{0.062(20)} = 16.3 \text{ Mte}(\text{S})$$

D. Outline the possible environmental effects of sulfur emissions. Be as comprehensive as possible. What considerations would be important in determining the actual consequences of sulfur emissions for human and environmental health in India? (10 points)

Environmental effects of sulfur emissions include: (a) air pollution; (b) acid deposition; (c) fertilization; (d) increased albedo. These could lead to (a) respiratory illness in humans; (b) acidification of surface waters and consequent elimination of certain aquatic species, damage to plants (via dry deposition on leaves, mobilization of toxic metals, and/or leaching of nutrients), damage to human health (via mobilization of toxic metals), and damage to structures containing carbonates; (c) increased plant growth; (d) decreased temperature. The considerations that are important in determining the magnitude of actual impacts are (a) the height of power-plant stacks and the proximity of human populations to power plants; (b) the buffering capacity and toxic-metal concentrations of soils where the acid is deposited, the sensitivity of animal and plant species in these areas, and the composition of water distribution pipes and the possible addition of buffering agents to the water supply; (c) whether sulfur is a limiting

nutrient in the ecosystem; and (d) a host of factors, including the concentrations of other climate-modifying substances in the atmosphere and the sensitivity of the regional climate to changes in radiative forcing.

2. The population of Belgium is about 10 million. The population growth rate is very close to zero. Using stock-flow-residence time considerations, estimate the number births and deaths in Belgium each year. (5 points)

Since the growth rate is nearly zero, we are almost in equilibrium, in which case birth rate = death rate = stock ÷ residence time. The residence time of a human is life expectancy, which is about 76 years in Belgium. (Anything in the 70 to 80 year range would be fine.) Thus:

$$\text{birth rate} = \text{death rate} = (10 \text{ million}) \div (76 \text{ years}) = 130,000/\text{yr}$$

The actual birth and death rates are 121,000 per year and 115,000 per year, respectively. The differences is due to immigration, gradually increasing life expectancy, and a slightly non-zero and time-varying growth rate.

3. Fluorescent lighting is much more efficient than incandescent, using one-fourth the electricity to deliver the same amount of light. In the U.S., about 7 percent of electricity is used for incandescent lighting. Electricity production is responsible for 35 percent of U.S. carbon emissions. If all incandescent lights were replaced by fluorescents, by about what percentage would U.S. emissions of carbon dioxide be reduced? (5 points)

Since fluorescents use one-fourth the electricity, we would save 75 percent of the electricity used for incandescent lighting, or $(0.75)(0.07) = 5.25$ percent of the electricity used for incandescent lighting. Thus, we would eliminate 5.25 percent of the carbon emissions due to electricity production, or $(0.0525)(0.35) = 1.84$ percent of total U.S. carbon emissions. In other words, about 2 percent.

4. Lake Morbid, in the coal-mining region of Lower Slobovia, has a surface area of 500 hectares and an average depth of 20 meters. The lake is the source of Coal Creek, which has an average flow at this point of 1.6 cubic meters per second. The surrounding mines discharge an average of 1 tonne per day of mine acid (i.e., sulfuric acid, H_2SO_4) into the lake.

- A. What is the flow rate of acid into the lake in moles per year? (5 pts)

$$F_{\text{H}_2\text{SO}_4} = \left[\frac{1 \text{ te}}{\text{day}} \right] \left[\frac{365 \text{ days}}{\text{yr}} \right] \left[\frac{10^6 \text{ g}}{\text{te}} \right] \left[\frac{\text{mole}(\text{H}_2\text{SO}_4)}{2(1) + 32 + 4(16) \text{ g}} \right] = 3.7 \cdot 10^6 \frac{\text{mole}(\text{H}_2\text{SO}_4)}{\text{yr}}$$

- B. Calculate the stock of acid in the lake, in moles. What assumption did you make to do this calculation? (5 points)

Assume that the residence time of acid in the lake is the same as the residence time of water in the lake (i.e., the acid is well-mixed); then

$$\tau = \frac{V}{F_{\text{H}_2\text{O}}} = \frac{(500 \text{ ha}) \left(10^4 \frac{\text{m}^2}{\text{ha}} \right) (20 \text{ m})}{\left(1.6 \frac{\text{m}}{\text{s}} \right) \left(3.15 \cdot 10^7 \frac{\text{s}}{\text{yr}} \right)} = 2.0 \text{ yr}$$

$$S_{\text{H}_2\text{SO}_4} = \tau F_{\text{H}_2\text{SO}_4} = (2.0 \text{ yr})(3.7 \cdot 10^6 \text{ mole/yr}) = 7.4 \cdot 10^6 \text{ mole}(\text{H}_2\text{SO}_4)$$

- C. Estimate the pH of the lake water, assuming that the mine acid is the only source of H^+ ion. (5 points)

$$[\text{H}^+] = \frac{2 \cdot S_{\text{H}_2\text{SO}_4}}{V} = \frac{2 \cdot (7.4 \cdot 10^6 \text{ mole})}{(10^8 \text{ m}^3)(10^3 \text{ L/m}^3)} = 1.5 \cdot 10^{-4}$$

$$\text{pH} = -\log_{10}[\text{H}^+] = -\log_{10}(1.5 \cdot 10^{-4}) = 3.8$$

5. Some have suggested using liquid fuels derived from biomass as substitutes for fuels derived from oil. In the U.S., the highest yielding crop is sorghum—about 19 tons of dry biomass per hectare per year. The energy content is about 17.5 MJ per kilogram of dry biomass, 90 percent of which can be captured in the form of ethanol.
- A. How much land would be required to offset U.S. oil imports, which in 1993 totaled 2,457 million barrels (Mbbbl)? How does this compare with the total amount of cropland harvested (300 million acres)? The energy content of petroleum is 6.1 GJ/bbl. (10 points)

$$\left[\frac{2457 \cdot 10^6 \text{ bbl}}{\text{yr}} \right] \left[\frac{6.1 \text{ GJ}}{\text{bbl}} \right] \left[\frac{\text{te}}{17.5 \cdot 0.9 \text{ GJ}} \right] \left[\frac{\text{ha} \cdot \text{yr}}{19 \text{ te}} \right] \left[\frac{2.47 \text{ acres}}{\text{ha}} \right] = 1.2 \cdot 10^8 \text{ acres}$$

About 120 million acres, or 40% of U.S. cropland that is harvested.

- B. The market price of sorghum is about ten cents per pound. Translate this into dollars per gigajoule, and compare it to the current price of oil (about \$20 per barrel). (10 points)

$$\left[\frac{\$0.1}{\text{lb}} \right] \left[\frac{2.2 \text{ lb}}{\text{kg}} \right] \left[\frac{\text{kg}}{15.8 \text{ MJ}} \right] \left[\frac{10^3 \text{ MJ}}{\text{GJ}} \right] = \$14/\text{GJ}$$

Compare to oil at \$20/bbl :

$$\left[\frac{\$20}{\text{bbl}} \right] \left[\frac{\text{bbl}}{6.1 \text{ GJ}} \right] = \$3.30/\text{GJ}$$

Sorghum is more than four times as expensive as oil per unit energy.

6. One of the complications of the climate-change problem is that so many substances released by humans influence global climate.

A. List the emissions that influence climate and list the human activities that give rise to these emissions. (5 points)

Carbon dioxide: fossil-fuel burning; biomass clearing; cement manufacture

Methane: natural gas leakage; livestock; rice growing

Nitrous oxide: overapplication of nitrogen fertilizers

Halocarbons: industrial use of CFCs, CFC-substitutes, and certain solvents

Ozone: created in urban air from NO_x, hydrocarbons, and sunlight

Aerosols: coal and biomass burning

B. The effect of emissions on climate is often measured in terms of their “instantaneous radiative forcing.” What is this? Why is radiative forcing an inadequate basis for comparing the potential climate effects of different types of emissions? (5 points)

Instantaneous radiative forcing is the effect that a given amount of gas or aerosol would have on the radiation balance of the Earth, if it were added to the atmosphere all at once. For example, doubling CO₂ would result in a forcing of 4.4 W/m²; which means that if CO₂ was doubled instantly, the Earth would radiate 4.4 fewer watts of energy back to space than it receives from the Sun.

Instantaneous radiative forcing is an inadequate basis for comparing emissions because substances have very different residence times in the atmosphere. The residence time of CO₂ is roughly ten times longer than CH₄, so that even if the radiative forcing of CO₂ and CH₄ emissions were equal, the warming effects of CO₂ would remain with us much longer than those of CH₄.

C. Suppose that the radiative forcing due to human interference with the atmosphere is known precisely. Describe the uncertainties in translating this into a description of how climate would change. (10 pts)

The uncertainties are due primarily to uncertainties in feedbacks in the global climate system. Without these uncertainties, translating a certain radiative forcing into a change in temperature would be straightforward. But as the Earth warms, the climate system adjusts in many ways to offset or amplify the temperature increase. For example, the concentration of water vapor may increase, trapping more infrared radiation; cloud cover, type, or altitude may change, altering the reflection of incoming solar and the trapping of outgoing infrared; ice and snow may melt, changing the albedo of the Earth; the rate of biological decay may increase, releasing additional

methane or carbon dioxide into the atmosphere; or the rate of biological growth may increase, drawing carbon out of the air. The effects vary over the surface of the Earth in ways that cannot be predicted in detail.

There is far more to climate and climate change than increasing temperature, however. Plants are affected by changes in precipitation, soil moisture, sunlight/cloudiness, and the length of the growing season. Climate models do not predict these changes very well. Even more important than changes in average values may be changes in the variance: floods and droughts, storms, periods of unusually hot or cold weather, etc. Changes in ocean currents may be very important.

7. The population of China continues to increase, even though the birth rate is decreasing. Also, the concentrations of CFCs in the atmosphere continue to increase, even though the rate of emission has decreased substantially. Why? Feel free to use drawings to illustrate your point. (10 points)

In both cases, the stocks (people in the first example, CFCs in the second) are not in equilibrium. In equilibrium, the flow into the system equals the flow out. In the first example, the birth rate would equal the death rate; in the second example, the rate of emission of CFCs would equal the rate of destruction in the stratosphere. Once a constant flow into the system is established, it takes a period of time to establish an equilibrium stock. In the first case, the outflow (death) is proportional to the number of old people, so we have to wait until the population of older people is constant (at least 70 years); in the second case, the outflow (stratospheric destruction) is proportional to the stock, so we have to wait for several times the residence time of CFCs, or on the order of one hundred years. After an equilibrium is established, a decrease in inflow will cause a decrease in the stock.

If a stock is not in equilibrium because it has been growing rapidly, then the outflow will be smaller than the inflow; if the residence time is long, as it is in these examples, the outflow can be much smaller than the inflow. Thus, unless the inflow is reduced to be equal to the current outflow, the population will continue to grow.

You could, of course, use a box model and equations to demonstrate this point, but words alone are just fine.