

FINAL EXAM SOLUTIONS

1. The population of Guatemala grew from 3.0 to 12.0 million from 1950 to 2000.

A. What was the average growth rate over this period? (3 points)

$$\left(\frac{12}{3}\right)^{\frac{1}{50}} - 1 = 0.0281 \cong 2.8\%/y \qquad \frac{\ln(12/3)}{50} = 0.0277 \cong 2.8\%/yr$$

B. The death rate averaged 7.3 deaths per year per 1000 people over this period. What was the average birth rate? (3 points)

$$d = 7.3/1000 = 0.0073/y$$

$$r = b - d \qquad b = r + d = 0.0277 + 0.0073 = 0.035 = 35 \text{ births/y per 1000}$$

C. If the population continued growing at the rate calculated in part A, in what year would it reach 24 million? (3 points)

$$T_{2x} = 0.69/r = 0.69/0.0277 = 25 \text{ y}$$

D. According to one scenario, the population of Guatemala will level off at about 30 million before 2050, when the average life expectancy will reach 75 years. Roughly how many children will be born in 2050 in this scenario? (3 points)

$$\text{In equilibrium (constant population), } b = d = S/\tau = (30 \cdot 10^6)/(75 \text{ y}) = 400,000/y$$

2. Someone wrote that “We could deal with the CO₂ problem quite easily by using it to soften water for agriculture. Water is hard in most of the U.S.; softening it would use all the CO₂ emitted by the burning of fossil fuels.” Evaluate this claim. (15 points)

“Hard” water contains high concentrations of dissolved calcium. Hardness is measured in grains per gallon (GPG) of CaCO₃; 1 grain = 65 mg. Soft water has less than 1 GPG; very hard water contains 100 GPG. Carbon dioxide can be used to precipitate the calcium as solid CaCO₃ (calcite). The U.S. emits 5.3 Gt/y of CO₂ from fossil-fuel burning and withdraws about 400 billion gallons of water per day.

(Ca in hard water is not from dissolved CaCO₃. Because the concentration of Ca is measured by precipitating it as CaCO₃, it is reported in terms of the amount of CaCO₃ that could be precipitated.)

Suppose all water withdrawn was very hard (100 GPG), and that CO₂ was used to remove all the calcium in the water:

$$\left[\frac{400 \cdot 10^9 \text{ gal}}{\text{d}} \right] \left[\frac{100 \text{ grains}_{\text{CaCO}_3}}{\text{gal}} \right] \left[\frac{0.065 \text{ g}}{\text{grain}} \right] \left[\frac{\text{mole}_{\text{CaCO}_3}}{(40 + 12 + 3 \cdot 16) \text{ g}} \right] \left[\frac{\text{mole}_{\text{CO}_2}}{\text{mole}_{\text{CaCO}_3}} \right]$$

$$\left[\frac{44 \text{ g}_{\text{CO}_2}}{\text{mole}_{\text{CO}_2}} \right] \left[\frac{\text{t}}{10^6 \text{ g}} \right] \left[\frac{\text{Gt}}{10^9 \text{ t}} \right] \left[\frac{365 \text{ d}}{\text{y}} \right] = 0.4 \frac{\text{Gt}_{\text{CO}_2}}{\text{y}}$$

The claim is dead wrong, particularly when you consider that not all water is very hard. Realistically, at most a few percent of U.S. CO₂ emissions could be sequestered in this way.

3. A recent report by the U.S. Global Change Research Program includes the following “key findings”: (a) temperatures will rise 5-10 °F over the next 100 years, with temperatures increasing more in the winter than the summer; (b) heavy and extreme precipitation events will become more frequent, yet some regions will get drier; (c) forest productivity is likely to increase over the next several decades, but possibly decrease over the longer term; (d) coastal wetland area will be reduced and coastal settlements will be at increased risk. Explain the origin of these findings. (15 points)

(a) Temperatures will rise 5–10 °F (3–6 °C) over the next 100 years as a result of increased concentrations of greenhouse gases—in particular, carbon dioxide. The equivalent CO₂ concentration will double over this period, resulting in a global average increase of 1.5–4.5 °C; the increase will be larger over the continental U.S. The temperature increase will be greater in the winter because of the increase in the greenhouse effect will be greater for cold, dry air (winter) than warm, moist air (summer).

(b) Heavy and extreme precipitation events will become more frequent because of an intensification of the hydrological cycle (higher surface temperatures result in increased evaporation and convection); some regions will get drier because the increased rainfall in that region will not compensate for the increase in evaporation.

(c) Forest productivity is likely to increase in the short term due to fertilization from increased CO₂ concentrations and nitrogen deposition, and increased rainfall, warmer temperatures, and a longer growth season. Over the longer term, species will find themselves in climates for which they are not well-adapted, and natural migration to more favorable climate may be slow, leading to forest die-back without compensating increases in forest cover.

(d) Coastal wetland area will be reduced due to sea level rise. New wetlands are unlikely to compensate for wetland loss, since humans will act to prevent the loss of existing land. Coastal settlements will be at increased risk due to the combined effects of sea-level rise and the increased frequency of storms.

4. The Three Gorges Dam, which is currently being constructed in China, will provide about 18,000 MW of hydroelectric power after it is completed. The dam has been very controversial; the reservoir will flood important historical artifacts and structures, and it will displace between one and two million people.

The alternative would have been coal-burning power plants. Estimate the carbon emissions avoided by Three Gorges. Assume that coal is 75% carbon, releases 28 GJ of heat per ton, and a net efficiency of 30 percent. If the “externality cost” of carbon emissions is \$100/tC, what is the corresponding benefit of Three Gorges per displaced person? (15 points)

$$[18 \text{ GW}_e] \left[\frac{\text{GJ}_e}{\text{GW}_e \text{ s}} \right] \left[\frac{\text{GJ}_t}{0.3 \text{ GJ}_e} \right] \left[\frac{t_{\text{coal}}}{28 \text{ GJ}_t} \right] \left[\frac{0.75 \text{ tC}}{t_{\text{coal}}} \right] \left[\frac{3.15 \cdot 10^7 \text{ s}}{y} \right] \left[\frac{\text{Mt}}{10^6 \text{ t}} \right] = 50 \frac{\text{MtC}}{y}$$

This is almost 1 percent of global emissions from fossil-fuel burning!

$$\left[\frac{\$100}{\text{tC}} \right] \left[\frac{50 \cdot 10^6 \text{ tC}}{y} \right] \left[\frac{1}{1 \text{ to } 2 \text{ million people}} \right] = \frac{\$2500 \text{ to } 5000}{\text{person} \cdot y}$$

Much greater than the income of these displaced people. Of course, the benefits accrue to everyone, while the costs are borne largely by the displaced people. Also, we didn't include the environmental costs of dam construction.

5. Why did the ozone hole form over Antarctica and not elsewhere? In other words, why did ozone depletion manifest itself as a hole in that particular region rather than as a thinning in the global ozone shield overall? If ozone depletion is confined to this completely unpopulated area, why were we so concerned about it? (10 points)

The ozone hole formed over Antarctica due to the unique conditions that exist during the Antarctic winter. First, an intense circumpolar vortex forms, confining and isolating the air above Antarctica from the rest of the atmosphere. Second, the natural production of ozone stops during winter, since the pole receives no sunshine. Third, the temperature becomes extremely low, leading to the formation of ice crystals even in this extremely dry air. The chlorine becomes adsorbed onto the surfaces of these crystals in the form of a weakly-bound molecule. In the spring, sunshine liberates the chlorine, which rapidly destroys ozone in a catalytic, heterogeneous reaction. (Heterogeneous refers to the fact that these reactions happen on the surfaces of ice crystals, rather than in the gas phase.)

It is worth noting that there has been a thinning in the global ozone layer—ozone depletion averages a few percent over the northern hemisphere. This thinning is very small in magnitude but more constant compared with the very deep but transient reduction in Antarctic ozone.

Even though Antarctica isn't populated, we're concerned because: (a) fisheries and other wildlife in the Antarctic may be affected; (b) the hole

extends over populated regions of Australia and South America; (c) a similar hole might form of the far-more-densely populated northern polar region; (d) the hole demonstrated quite vividly our ignorance of atmospheric chemistry, and we can't rule out more surprises; (e) the same compounds that cause the Antarctic hole—CFCs—also are thought to be responsible for the more global thinning of the ozone layer.

6. The wind turbines at Carmarthen Bay, Wales, cost \$500,000 fully installed, have a rotor diameter of 22 m, and produce up to 300 kW of electricity. Of course, the wind speed is not always high enough to allow the turbines to produce 300 kW; the capacity factor (average output divided by maximum output) is 0.38. Maintenance and operations costs average about \$5,000/y per turbine.

- A. Very roughly, what is the cost of the electricity produced by these turbines? Assume a capital charge factor of 10 percent per year. (10 points)

$$\frac{[\$500,000] \left[\frac{0.1}{y} \right] + \left[\frac{\$5,000}{y} \right]}{[300 \text{ kW}_e] \left[\frac{8760 \text{ h}}{y} \right]} = \frac{\left[\frac{\$55,000}{y} \right]}{\left[\frac{10^6 \text{ kWh}}{y} \right]} = \frac{\$0.055}{\text{kWh}}$$

- B. The center of the rotor is 22 m above the ground. Compare the height of the turbine with the height of Van Munching Hall. (3 points)

Van Munching Hall is about 25 m tall (about 5 stories including the roof/clock, at 5 m per story). The turbine would be a total of 33 m tall. A correct answer would be taller, but not twice as tall.

7. In 1990, wet deposition of anions in the eastern U.S. (east of the Mississippi River) averaged 22.06 kg/ha of SO_4^{2-} and 14.18 kg/ha of NO_3^- . Average precipitation was 122.7 cm.

- A. Where did most of this sulfate and nitrate come from? (5 points)

Sulfate: coal burning; oil burning; smelting; natural emissions (biological sources, volcanoes, and sea spray)

Nitrate: automobiles, power plants, natural emissions (lightening)

- B. Characterize the relative importance of SO_2 and NO_x , as contributors to acid rain. (5 points)

$$\left[\frac{22.06 \text{ kg}_{\text{SO}_4^{2-}}}{\text{ha} \cdot \text{y}} \right] \left[\frac{1000 \text{ g}}{\text{kg}} \right] \left[\frac{\text{mole}_{\text{SO}_4^{2-}}}{96 \text{ g}} \right] \left[\frac{2 \text{ mole}_{\text{H}^+}}{\text{mole}_{\text{SO}_4^{2-}}} \right] \cong 460 \frac{\text{mole}_{\text{H}^+}}{\text{ha} \cdot \text{y}}$$

$$\left[\frac{14.18 \text{ kg}_{\text{NO}_3^-}}{\text{ha} \cdot \text{y}} \right] \left[\frac{1000 \text{ g}}{\text{kg}} \right] \left[\frac{\text{mole}_{\text{NO}_3^-}}{62 \text{ g}} \right] \left[\frac{1 \text{ mole}_{\text{H}^+}}{\text{mole}_{\text{NO}_3^-}} \right] \cong 230 \frac{\text{mole}_{\text{H}^+}}{\text{ha} \cdot \text{y}}$$

So SO_2 is roughly twice as important as NO_x .

- C. Ignoring the presence of other anions and cations (other than H^+), what was the average pH of rain? (5 points)

$$[\text{H}^+] = \left[\frac{690 \text{ mole}_{\text{H}^+}}{\text{ha} \cdot \text{y}} \right] \left[\frac{\text{ha}}{10^4 \text{ m}^2} \right] \left[\frac{\text{y}}{1.227 \text{ m}} \right] \left[\frac{\text{m}^3}{10^3 \text{ L}} \right] = 5.6 \cdot 10^{-5} \bar{\text{M}}$$

$$\text{pH} = -\log_{10} [\text{H}^+] = -\log_{10} [5.6 \cdot 10^{-5}] = 4.25$$

- D. Aside from acid rain, what other environmental effects do SO_2 and NO_x emissions have? (5 points)

**SO_2 : regional cooling due to albedo effect of sulfate aerosols
air pollution—human health hazard**

NO_x : fertilization of forests and other terrestrial ecosystems, resulting in enhanced carbon sequestration

eutrophication of aquatic ecosystems

air pollution—human health hazard

increased rate of denitrification and production of N_2O —a greenhouse gas and ozone-depleting compound